Section One: Calculator – free (50 marks)

This section has **six (6)** questions. Answer all questions. Write your answers in the spaces provided.

Working time: 50 minutes

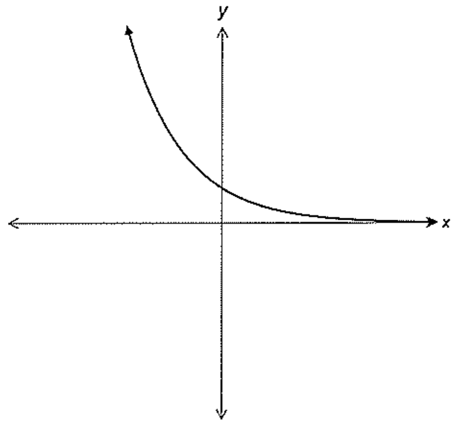
Question 1 (7 marks)

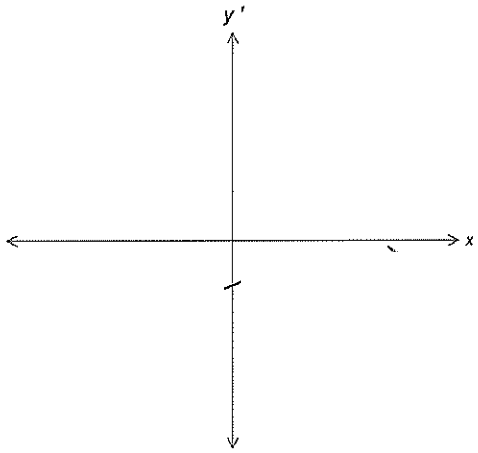
(a) Simplify $\dfrac{3}{x^2-4} - \dfrac{5}{x+2}$ (3)

(b) Simplify $\dfrac{3m^2-6m-24}{m^2-5m+4} \div \dfrac{m^2-m-6}{m^2-3m}$ (4)

Question 2 **(9 marks)**

(a) Sketch the graph of the derivative function (for the function shown) on the axes provided. (2)





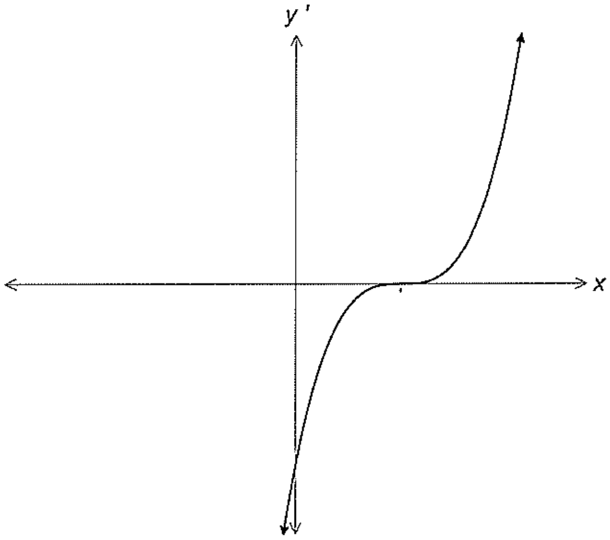
(b) Differentiate the following with respect to *x*.

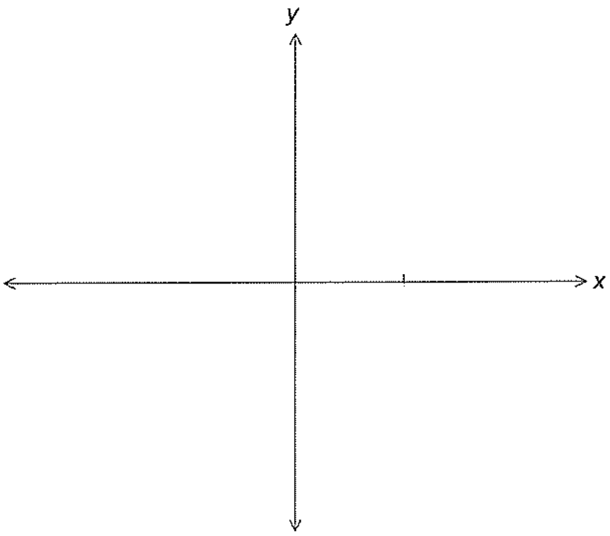
(i) $f(x) = \dfrac{-x}{x^2+1}$ (express in simplest form) (3)

(ii) $g(x) = (x+1)^2 e^{x^2}$ (do not simplify) (2)

Question 2 (continued)

(c) Given the derivative function, sketch the graph of the function on the axes provided. (2)





Question 3 **(12 marks)**

Consider the curve $y = x^3 - 2x^2 - 4x + 3$

(a) It is claimed that the tangent to the curve at the point where $x = 1$ passes through the point (3, 8). Is this claim valid? Justify your answer. (4)

(b) Determine the value of x for which y is a maximum. (4)

Question 3 (continued)

(c) Solve the following system of equations. (4)

$$x - 2y + 4z = 2$$
$$2x + y + 3z = -1$$
$$-x - y - 2z = 0$$

Question 4 **(10 marks)**

(a) Determine c given that the graph of $f(x)=cx^2+x^{-2}$ has a point of inflection at $(1,\ f(1))$. (3)

Question 4 (continued)

(b) The functions $f(x)$ and $g(x)$ are defined as follows
$f(x) = x^2 - 4$ and $g(x) = \sqrt{x-5}$

(i) Determine the simplified expressions for $f[g(x)]$ and $g[f(x)]$. (3)

(ii) Determine the range of $f[g(x)]$. (2)

(iii) Determine the domain of $g[f(x)]$. (2)

Question 5 **(7 marks)**

(a) Determine $\int (1+3x^2)(x-2)\,dx$ (3)

(b) Determine $\int 4x^3(3x^4-5)^7\,dx$ (2)

(c) Determine $\int 12x e^{x^2}\,dx$ (2)

Question 6 **(5 marks)**

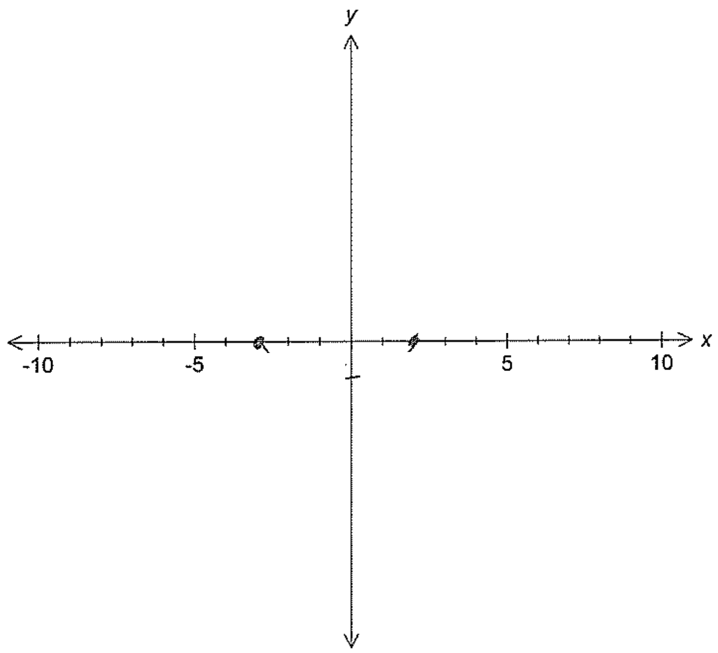
$f(x)$ is defined such that $\int_{-3}^{6} f(x)\,dx = 24$ and $\int_{2}^{6} f(x)\,dx = 36$

(a) Find

(i) $\int_{-3}^{2} f(x)dx$ (1)

(ii) $\int_{-3}^{2} \left(4f(x)+3\right)\,dx$ (3)

(b) Sketch a possible graph of *y=f(x)* for $-3 \le x \le 6$. Your graph should display the relative areas of important regions but you do not need to draw this graph to scale. (1)



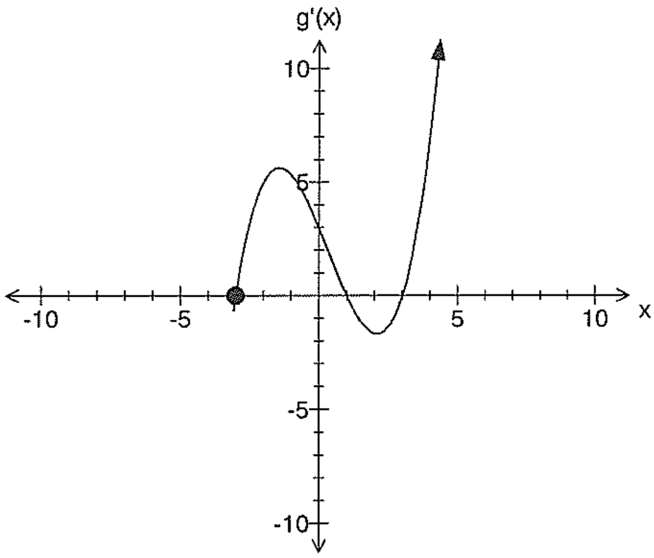
Section Two: Calculator – assumed **(100 marks)**

This section has **ten (10)** questions. Answer all questions. Write your answers in the spaces provided.

Working time: 100 minutes

Question 7 **(7 marks)**

The graph of g′(x) is given below.



(a) What can be said about the gradient of the function $g(x)$ between x = –3 to x = 1? (1)

(b) When does the function, $g(x)$ have a negative gradient? (2)

(c) State an equation for the tangent to the graph of $g(x)$ at x = 3. (1)

(d) Find the value of x at which $g(x)$ has a relative maximum for $-3 \le x \le 4$ (1)

(e) Find the x-coordinate of each point of inflection of the graph of $g(x)$ for $-3 \le x \le 4$ (2)

Question 8 **(12 marks)**

(a) Events A and B are such $P(A)=\frac{1}{2}$, $P(B)=\frac{7}{12}$ and $P(\overline{A \cup B})=\frac{1}{4}$

Show that events A and B are not mutually exclusive. (3)

(b) A toy robot has 3 main components (X, Y and Z) which are manufactured separately and then assembled together. Previous random testing of components has shown that:

P (X defective) = 0.002, P (Y defective) = 0.015, P (Z defective) = 0.003

If a toy robot is selected at random, what is the probability that:

(i) only component Y is defective, (2)

(ii) at least one of its components are defective. (2)

Question 8 (continued)

(c) If X and Y are independent events and $P(X) = 0.75$ and $P(X \cup Y) = 0.875$, find

(i) $P(Y)$ (3)

(ii) $P(Y \mid X)$ (1)

(iii) $P(X \mid Y')$ (1)

Question 9 **(12 marks)**

(a) It takes 12 hours to drain a storage tank by opening the valve at the bottom. The depth, y, of fluid in the tank t hours after the valve is opened is given by

$y = 6\left(1 - \frac{t}{12}\right)^2$ metres.

(i) Find the rate $\frac{dy}{dt}$ m/hour at which the tank is draining at time, t. (2)

(ii) When is the fluid in the tank falling fastest and slowest?

What are the values of $\frac{dy}{dt}$ at these times? (3)

Question 9 (continued)

(c) If $y = kx^3$ for some constant k, use the incremental formula to establish the percentage increase in x required to yield a 15% increase in y. (3)

(d) A company sells goods such that its revenue, in dollars, from selling x items is given by the equation,

$$R(x) = 5x(20x - x^2)$$

(i) Determine the marginal revenue when $x = 10$. (2)

(ii) What does marginal revenue represent? (2)

Question 10 **(7 marks)**

The Australian Kayak team must select 4 elite rowers from 14 possible contenders to be the new 'Awesome Foursome'.

(a) How many different selections are possible? (1)

Mike is the singles kayak champion and Geoff is the runner up champion.

(b) What is the probability that of the 4 rowers chosen at random:

(i) Mike is included? (1)

(ii) Mike and Geoff are included? (1)

(iii) Mike or Geoff is selected? (2)

(c) If Mike is selected for the Kayak team, what is the probability that Geoff is also selected? (2)

Question 11 **(12 marks)**

(a) The function $f(x)$ is differentiable for all $x \in R$ and satisfies the conditions

$f'(x) < 0$ where $x < 2$

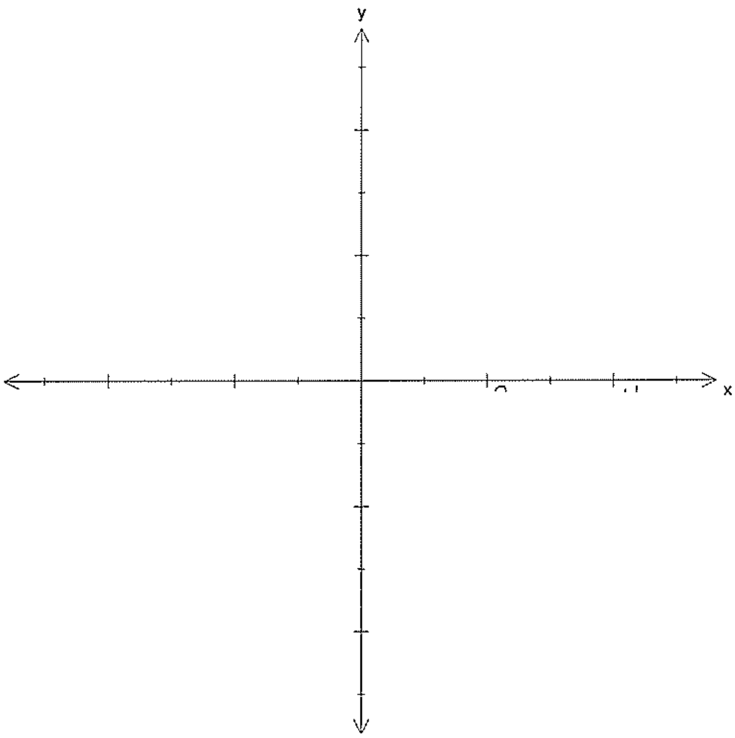
$f'(x) = 0$ where $x = 2$

$f'(x) = 0$ where $x = 4$

$f'(x) > 0$ where $2 < x < 4$

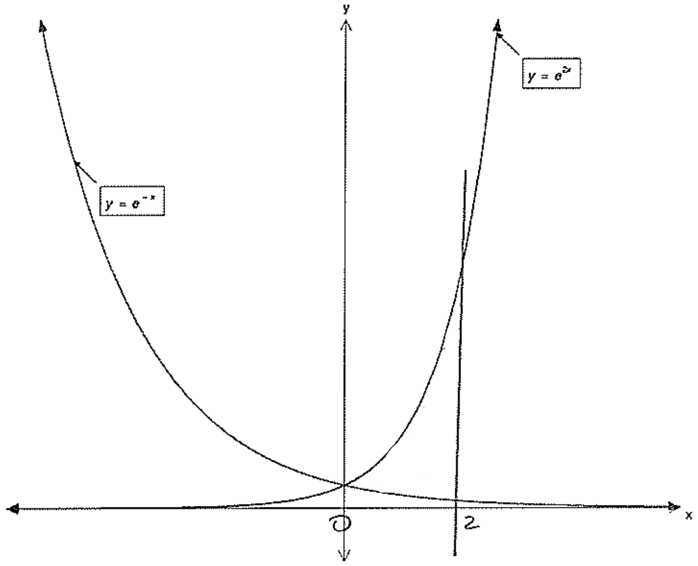
$f'(x) > 0$ where $x > 4$

(i) Draw a sketch of this function $f(x)$. (3)



Question 11 (continued)

(b) The curve $y=e^{2x}$ and $y=e^{-x}$ intersect at the point (0, 1) as shown in the diagram.



Find the area enclosed by the curves and the line $x=2$.
Leave your answer in terms of e. (4)

Question 11 (continued)

(c) State the resulting equation when the graph of $y = e^x$ undergoes the following transformations **in succession**: (3)

- horizontal dilation of factor $\frac{1}{3}$
- reflection about the y-axis
- vertical translation 5 units in the direction of the negative y-axis
- horizontal translation 3 units in the direction of the positive x-axis
- vertical dilation of factor 2

(d) The point $(3, 0.5e^4)$ lies on the curve of $y = 0.5e^{x+1}$. Identify the subsequent location of this point if the transformations listed below are applied **in succession**. (2)

- reflection about the x-axis
- horizontal translation 7 units in the direction of the negative x-axis
- vertical translation 3 units in the direction of the positive y-axis
- reflection about the y-axis

Question 12 **(9 marks)**

Research has been conducted to determine the benefits of a flu vaccine before winter for adults over 65. The following information has been obtained:

60% of the target population (i.e. adults over 65) had the flu vaccine and of these 22% actually developed the flu, 3% developed a chest infection and the remainder had no flu-like symptoms over the winter.

Of those who did not have the flu vaccine 12% developed a chest infection.

The proportion of those studied who did not have the vaccine and had no flu-like symptoms over the winter was 0.096.

14% of all those who developed a chest infection also got pneumonia.

(Note that in this same sample no one developed both the flu and a chest infection)

(a) Draw a tree diagram to represent the above information. (4)

(b) For a randomly chosen person from this study determine the probability that:

(i) the person developed the flu if they did not have the flu vaccine. (1)

(ii) the person had the flu vaccine and developed pneumonia. (1)

(iii) the person had the vaccine if they developed pneumonia. (3)

Question 13 **(12 marks)**

Consider the letters of the word POLICE.

How many arrangements are there of these 6 letters (without repetition) if

(a) each arrangement must end with a vowel? (1)

(b) the vowels in each arrangement must be together? (2)

(c) the vowels must be separated by the consonants (ie 2 vowels must not be together)? (2)

Now suppose that 4 letters are chosen from this word and that the **order of selection is unimportant.**

(d) How many different 4 letter groups are possible if

(i) there is no restriction? (1)

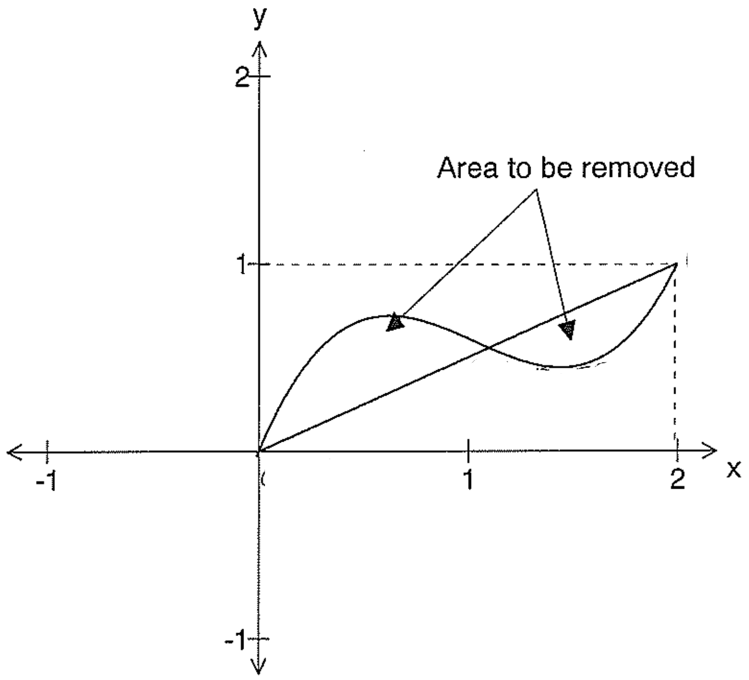
(ii) there must be at least one vowel? (2)

(e) Determine the probability that in the four letter selection that is made, whenever O appears, E also appears. (4)

Question 14 **(11 marks)**

(a) A dressmaker wishes to cut a section of cloth from a piece of material measuring 2 metres by one metre. The curved edges of the piece of cloth to be removed are defined as being between the following equations:

$$y_1 = 0.5x \text{ and } y_2 = x^3 - 3.1x^2 + 2.7x$$



(i) Label the 3 points of intersection with co-ordinates. (1)

(ii) Write an integral that would give the area of region bound by the two functions. (2)

(iii) Calculate the area of the cloth removed, correct to 2 decimal places. (2)

Question 14 (continued)

(b) A group of anthropologists found that human tooth size is continuing to decrease, such that $\frac{dS}{dt} = kS$. In Northern Europeans, for example, tooth size reduction now has a rate of 1% per 1000 years.

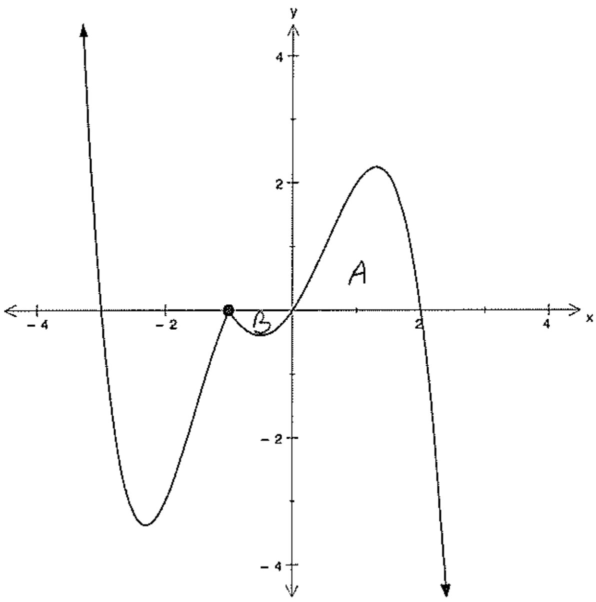
(i) If ***t*** represents time in years and ***S*** represents tooth size, find the value of **k**, rounded to 8 decimal places. (2)

(ii) In how many years will human tooth size be 90% of their present size? (2)

(iii) What will be our descendant's tooth size 20 000 years from now? (as a percentage of our present tooth size) (2)

Question 15 **(8 marks)**

(a) For the function $y = f(x)$ below



It is known that

$$\int_{-3}^{-1} f(x)\,dx = -75$$

$$\int_{-1}^{2} f(x)\,dx = 20$$

The area between the curve and the x-axis from $x = -1$ to $x = 2$ is 80 square units.

Use the information above and mathematical reasoning to determine the value of each of the following.

(i) $\int_{-1}^{0} f(x)\,dx$ (3)

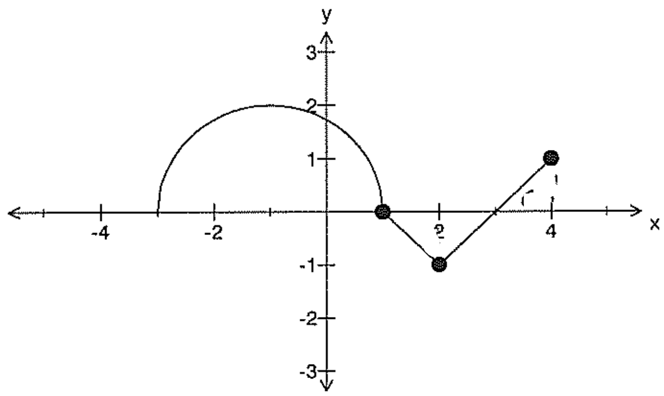
(ii) the area between the curve and the x-axis from $x = -3$ to $x = 0$ (1)

Question 15 (continued)

(iii) $\int_{-3}^{2} f(x)\,dx$ (1)

(b) The graph of a function $f(x)$ consists of a semi- circle and two line segments as shown.

Find the exact value of $\int_{-3}^{4} f(x)\,dx$ (3)



Question 16 **(10 marks)**

A piece of wire 8cm long is cut into two unequal parts. One part is used to form a rectangle that has a length three times its width. The other part of the wire is used to form a square.

(a) If the width of the rectangle is x units, show that the equation that will give the sum of the areas of the rectangle and the square in terms of x is: (5)

$$A = 7x^2 - 8x + 4$$

(b) Using Calculus, find the length of each part of the wire when the sum of the areas is a minimum. (5)